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STABILIZATION OF TRANSLATIONAL MOTION OF A CONTROLLED RIGID BODY

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Liapunov's vector function method is used to obtain the sufficient conditions of stability " in the large" of a position of equilibrium of a controlled rigid body relating to its center of mass, and of a specified direction of the mass center velocity vector.

Let us consider a rigid body m and the principal central moments of inertia A, B, and C, moving through the atmosphere. Let v denote the velocity of the center of mass of the body and p, q, r be the projections of the instantaneous angular velocity of rotation of the body onto the principal central axes of inertia C_x, C_y and C_z respectively. The dynamic equations of the translational rotational motion of the body written in terms of the projections on these axes, will have the following form:

$$m(v_x^{\cdot} + qv_z - rv_y) = \Sigma F_{ix} \quad (xyz,\ pqr) \tag{1}$$
$$Ap^{\cdot} + (C - B)qr = \Sigma M_{ix} \quad (xyz,\ pqr,\ ABC)$$

Here in the right-hand sides we have the sum of the projections of the forces and moments on the principal central axes of inertia of the body; symbols within the brackets indicate that the remaining equations are obtainable by cyclic permutation of the indices. The body is acted upon by the following forces: the thrust determined by the engine mode of operation, the motion of the center of mass and by the atmospheric conditions; the force of gravity - the aerodynamic forces depending on the geometry of the body and on the kinematic parameters of its motion i. e. the velocity of the center of mass, the angle of attack α and side slip angle β; and finally the controlling forces generated by the controls (this is done by deflecting the control surfaces from a certain position by the angles δ_x, δ_y and δ_z [1, 2].

The principal moment of the external forces acting on the body consists of a static moment which depends on the position of the axes of inertia relative to the mass center velocity vector (angles α and β), the aerodynamic damping moment which depends on the angular velocity of rotation of the body $(p, q, r, \alpha^{\cdot}, \beta^{\cdot})$, and the control moment originated by the control system $(\delta_x, \delta_y, \delta_z)$ [1, 2]. Moreover, all the above moments depend on the flight mode (the mass center velocity, and the atmospheric conditions at the given altitude).

The above statements concerning the forces and moments acting on the body show us the necessity of passing from the parameters v_x, v_y and v_z to the quantities v, α and β. The direction of the mass center velocity vector is defined by the angles α and β relative to the axis C_x in the C_{xz} and C_{xy}, planes, then the formulas of the above change on parameters for small values of the angles of attack and side-slip angle have the form [1]

$$v_x = v, \quad v_y = -v_\alpha, \quad v_z = v_\beta \tag{2}$$

Let us pose a problem of stability in the large of the motion of the body moving at a velocity v_0 , of the form

$$p = q = r = 0, \quad \alpha = \alpha_0, \quad \beta = \beta_0 \tag{3}$$

i. e. the problem of determining the conditions and controls which will have to be imposed on the parameters of the body, to make perturbations in the quantities p, q, r, v, α and β to tend asymptotically to their values given by (3), irrespective of their initial magnitude.

Assuming that in the perturbed motion

$$p \rightleftarrows p, \quad q \rightleftarrows q, \quad r \rightleftarrows r, \quad \alpha \to \alpha_0 + \alpha, \quad \beta \to \beta_0 + \beta \tag{4}$$

and noting that in the unperturbed motion (3) the sum of the projections of the forces and moments of forces on the principal central axes of inertia are equal to zero, we obtain the following result: in the perturbed motion the above quantities can be represented by series in powers of their perturbed values (4) beginning with the first power. It follows that for many aircraft we can write, in the perturbed motion [2]

$$\begin{aligned}
&\frac{\Sigma F_{iy}}{mv_0\alpha_0} = a_y\alpha + C_{xy}^{\cdot}\delta_x + F_{y2} \\
&\frac{\Sigma F_{iz}}{mv_0\beta_0} = a_z\beta + C_{zy}\delta_y + F_{z2} \\
&\frac{\Sigma M_{ix}}{A} = m_{xx}p + m_{xy}q + b_{xx}\delta_x + b_{xy}\delta_y + b_{x\beta}\beta + M_{x2} \\
&\frac{\Sigma M_{iy}}{B} = m_{yx}p + m_{yy}q + b_{yx}\delta_x + b_{yy}\delta_y + b_{y\beta}\beta + M_{y2} \\
&\frac{\Sigma M_{iz}}{C} = C_{z\alpha}\alpha + C_{z\alpha}\dot{\alpha} + m_{zz}r + b_{zz}\delta_z + M_{z2}
\end{aligned} \tag{5}$$

where $F_{y2}, F_{z2}, M_{x2}, M_{y2}$ and M_{z2} are functions of the variables p, q, r, α and β . The expansions of these functions into series in powers of the above variables begin with the second order terms. We shall no longer consider the first equation of (1), since it defines the ratio of the forces when the center of mass of the body moves at the velocity v_0, the thrust being predominant among these forces. Then the system (1) yields the following set of equations of perturbed motion:

$$\begin{aligned}
&\alpha^{\cdot} = a_{y\alpha}\alpha - r + C_{yx}\delta_x + F_{y2} \\
&\beta^{\cdot} = a_{z\beta}\beta + q + C_{zy}\delta_y + F_{z2} \\
&p^{\cdot} = m_{xx}p + m_{xy}q + b_{x\beta}\beta + b_{xx}\delta_x + b_{xy}\delta_y + M_{x2} \\
&q^{\cdot} = m_{yx}p + m_{yy}q + b_{y\beta}\beta + b_{yx}\delta_x + b_{yy}\delta_y + M_{y2}. \\
&r^{\cdot} = C_{z\alpha}\alpha + C_{z\alpha^{\cdot}}\alpha^{\cdot} = m_{zz}r + b_{zz}\delta_z + M_{z2}
\end{aligned} \tag{6}$$

The above set of equations of perturbed motion (6) of the proper rigid body, is supplemented by the equation of controls which have the form [3]

$$W_x\delta_x^{\cdot} + S_x\delta_x = f_x(\sigma_x, t)$$
$$T_x\sigma_x^{\cdot} + \sigma_x = a_{px}p + a_{\delta x}\delta_x$$

where $W_x, W_y, W_z, T_x, T_y, T_z, a_{px}, a_{qy}, a_{zz}, a_{\delta x}, a_{\delta y}, a_{\delta z}$ are constants and f_x, f_y, f_z are nonlinear functions satisfying the conditions

$$0 \leqslant \sigma_i f_i(\sigma_i, t) \leqslant k\sigma_i^2 \quad (i = xyz)$$

As the result Eqs. (6) and (7) of perturbed motion form a closed system. We use the Liapunov vector function [4] to obtain the sufficient conditions of stability in the large of the state of equlibrium for this system.

To solve the problem, we split the eleventh order system of Eqs. (6) and (7) into two subsystems. The first subsystem will consist of the set of equations (6) without the last terms in the right-hand sides, and the equations

$$\delta_i = -(S_i / W_i)\delta_i, \quad i = x, y, z$$

The second subsystem contains the equations

$$\sigma_i^{\cdot} = -\sigma_i / T_i, \quad i = x, y, z$$

The correlation vectors are

$$g_{12} = \left(F_{y2}, F_z, M_x, M_y, M_z^*, \frac{f_x(\sigma_x t)}{W_x}, \frac{f_y(\sigma_y t)}{W_y}, \frac{f_z(\sigma_z t)}{W_z}\right)$$
$$(M_z^* = M_z + C_{z\dot{\alpha}}F_y)$$
$$g_{21} = \left(\frac{a_{px}p + a_{\delta x}\delta_x}{T_x}, \frac{a_{qy}q + a_{\delta y}\delta_y}{T_y}, \frac{a_{rz}r + a_{\delta z}\delta_z}{T_z}\right)$$

The above linear subsystems are stable, and the scalar Liapunov functions V_1 and $V_2 = {}^1/_2(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$ which exist for these subsystems contain the Krasovskii constants C_{1k} $(k = 1, 2, 3, 4)$ and $C_{21} = C_{22} = {}^1/_2$, $C_{23} = \max\{1/T_x, 1/T_y, 1/T_z\}$, $C_{24} = 1$.

For g_{21}, we obviously have k_{21}, such that $\|g_{21}\| \leqslant k_{21}\|x_1\|$, where $x_1 = (\alpha, \beta, p, q, r, \delta_x, \delta_y, \delta_z)$. For g_{12} there exists k_{12}, such that $\|g_{12}\| \leqslant k_{12}\|x_2\|$, where $x_2 = (\sigma_x, \sigma_y, \sigma_z)$, under the condition that the nonlinear functions of forces and moments F and M satisfy estimates of the form

$$F_{ij} \leqslant l_{ij}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2), \quad M_{ij} \leqslant n_{ij}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$$

We shall assume that F and M satisfy these estimates.

Since the above subsystems are linear and exponentially stable and all conditions pf the Liapunov vector function theorem [4, 5] hold, the sufficient conditions of the asymptotic stability in the large of the initial nonlinear system (6), (7) will be given by the necessary and sufficient condition of stability of the constant matrix

$$A = \left\| \begin{matrix} -\dfrac{C_{13}}{2C_{12}} & \dfrac{C_{14}{}^2 k_{12}{}^2}{2C_{13}C_{21}} \\ \dfrac{C_{24}{}^2 k_{21}{}^2}{2C_{23}C_{11}} & -\dfrac{C_{23}}{2C_{22}} \end{matrix} \right\|$$

Thus, by virtue of the Routh-Hurwitz theorem, the condition of stability can be written in the form of an algebraic inequality

$$k_{12} < \sqrt{\frac{C_{23}C_{12}C_{22}}{C_{11}}\,\frac{C_{13}}{C_{14}C_{24}K_{21}}}$$

In this manner we have obtained the condition of asymptotic stability in the large of the initial nonlinear system (6), (7), in the form of the limit of k_{12}, i.e. in the form of an estimate of the nonlinear functions F and M.

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